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A NONLOCAL PROBLEM FOR PSEUDOHYPERBOLIC EQUATION

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In this paper we present some new results concerning with nonlocal problems for evolution equations. We consider a problem with nonlocal integral conditions for a pseudohyperbolic equation and prove the existence and uniqueness of a generalized solution to the problem. A proof is based on a priori estimates in Sobolev spaces and embedding theorems.

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В праці ми отримали деякі нові результати, що стосуються нелокальних задач для еволюційних рівнянь. Вивчена задача з нелокальними інтегральними умовами для псевдогіперболічного рівняння і доведено існування та єдиність узагальненого розв'язку. Доведення базується на апріорних оцінках в просторах Соболєва та теоремах вкладення.

Introduction

Pseudohyperbolic equations form important and interesting subclass of Sobolev type equations. Such equations may describe nonstationary waves in stratified and rotating liquid. The starting point in studying of Sobolev type equations is [1]. Now there are a lot of works devoted to initial and boundary value problems for Sobolev type equations (see [2] and references therein). One of recent works dealing with some problems for pseudohyperbolic equations is [3]. In these works authors study qualitative characteristics of solutions to initial-boundary value problems. On the other hand, various physical problems demand nonlocal conditions [4]. Recently, nonlocal boundary value problems with integral conditions have been actively studied [5], [6] (see also references therein). However the majority of the works deals with second-order equations.

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Motivated by this, we consider a nonlocal problem with integral conditions for a pseudo-hyperbolic equation.

Let $Q_T = (0,l) \times (0,T)$. Consider an equation

$$Lu \equiv \frac{\partial^2}{\partial t^2}(u - u_{xx}) - (a(x,t)u_x)_x + c(x,t)u = f(x,t). \tag{1}$$

and set a problem: find in $Q_T = (0,l) \times (0,T)$ a function u that is a solution of (1), satisfies initial data

$$u(x,0) = \varphi(x), \quad u_t(x,0) = \psi(x) \tag{2}$$

and nonlocal conditions

$$\begin{aligned} &\tfrac{\partial^2}{\partial t^2}u_x(0,t) + a(0,t)u_x(0,t) + \int_0^l K_1(x,t)u(x,t)dx = 0, \\ &\tfrac{\partial^2}{\partial t^2}u_x(l,t) + a(l,t)u_x(l,t) + \int_0^l K_2(x,t)u(x,t)dx = 0. \end{aligned} \tag{3}$$

Main results

At first we give a definition of a generalized solution to the problem using the standard method [7, p.92]. To this end we consider an identity

$$\int_0^T \int_0^l (Lu - f)v dxdt = 0,$$

where $u \in C^4(Q_T) \cap C^3(\bar{Q}_T)$ and satisfies (1)—(3), $v \in C^2(\bar{Q}_T)$, $v(x,T) = 0$. Integrating this identity by parts we obtain

$$\int_0^T \int_0^l (-u_t v_t - u_{xt}v_{xt} + au_x v_x + cuv)dxdt + \int_0^l u_t v\Big|_0^T dx +$$
$$+ \int_0^l u_{xt}v_x\Big|_0^T dx - \int_0^T (u_{xtt} + au_x)v\Big|_0^l dt = \int_0^T \int_0^l fv dxdt.$$

Note that $v(x,T) = 0$ implies $v_x(x,T) = 0$ and recall that the conditions (3) hold. Then the identity assumes the form

$$\int_0^T \int_0^l (-u_t v_t - u_{xt}v_{xt} + au_x v_x + cuv)dxdt - \int_0^T v(0,t)\int_0^l K_1(x,t)u(x,t)dxdt +$$
$$+ \int_0^T v(l,t)\int_0^l K_2(x,t)u(x,t)dxdt = \int_0^l [\psi(x)v(x,0) + \psi'(x)v_x(x,0)]dx + \int_0^T \int_0^l fv dxdt. \tag{4}$$

Introduce some notation. Let

$$W(Q_T) = \{u : u \in W_2^1(Q_T),\ u_{xt} \in L_2(Q_T)\}, \quad \hat{W}(Q_T) = \{v : v \in W(Q_T),\ v(x,T) = 0\}.$$

It is easy to see that the identity (4) makes sense for $u \in W(Q_T),\ v \in \hat{W}(Q_T)$ and may form a basis of the following definition.

Definition 1. *A function $u \in W(Q_T)$ is said to be a generalized solution to the problem (1)—(3) if $u(x,0) = \varphi(x)$ and for every $v \in \hat{W}(Q_T)$ the identity (4) holds.*

Theorem 1. *If $a \in C(\bar{Q}_T),\ a(x,t) \geq a_0 > 0,\ a_t \in C(\bar{Q}_T),\ c \in C(\bar{Q}_T),$
$\varphi \in W_2^1(0,l),\ \psi \in W_2^1(0,l),\ K_i \in C(\bar{Q}_T), f \in L_2(Q_T),$
then there exists a unique generalized solution to the problem (1)—(3).*

Proof. Uniqueness. Let u_1 and u_2 be two solutions to (1)—(3). Then $u = u_1 - u_2$ satisfies $u(x,0) = 0$ and the identity

$$\int_0^T\int_0^l (-u_t v_t - u_{xt}v_{xt} + au_x v_x + cuv)dxdt - \int_0^T v(0,t)\int_0^l K_1(x,t)u(x,t)dxdt +$$
$$+ \int_0^T v(l,t)\int_0^l K_2(x,t)u(x,t)dxdt = 0$$

holds for every $v \in \hat{W}(Q_T)$. We shall take v in this way:

$$v(x,t) = \begin{cases} \int_\tau^t u(x,\eta)d\eta, & 0 \leq t \leq \tau, \\ 0, & \tau \leq t \leq T. \end{cases}$$

After integrating by parts in the first term we obtain

$$\frac{1}{2}\int_0^l [v_t^2(x,\tau) + a(x,0)v_x^2(x,0) + v_{xt}^2(x,\tau)]dx =$$
$$= \int_0^\tau\int_0^l c(x,t)v(x,t)v_t(x,t)dxdt - \frac{1}{2}\int_0^\tau\int_0^l a_t(x,t)v_x^2(x,t)dxdt -$$
$$- \int_0^\tau v(0,t)\int_0^l K_1(x,t)v_t(x,t)dxdt + \int_0^\tau v(l,t)\int_0^l K_2(x,t)v_t(x,t)dxdt. \quad (5)$$

Our nearest aim is to derive an estimate of a right side of (5). To this end consider at first two terms generated by nonlocal conditions. By virtue of Cauchy inequality

$$\Big|\int_0^\tau v(0,t)\int_0^l K_1(x,t)v_t(x,t)dxdt\Big| \leq \frac{1}{2}\int_0^\tau v^2(0,t)dt + \frac{k_1}{2}\int_0^\tau\int_0^l v_t^2 dxdt,$$

$$\Big|\int_0^\tau v(l,t)\int_0^l K_2(x,t)v_t(x,t)dxdt\Big| \leq \frac{1}{2}\int_0^\tau v^2(l,t)dt + \frac{k_2}{2}\int_0^\tau\int_0^l v_t^2 dxdt,$$

where

$$k_i = \max_{[0,T]} \int_0^l K_i^2(x,t)dx, \quad i \in \{1,2\}.$$

Later on, we shall need the following inequalities

$$v^2(0,t) \le 2l \int_0^l v_x^2(x,t)dx + \frac{2}{l}\int_0^l v^2(x,t)dx, \quad v^2(l,t) \le 2l \int_0^l v_x^2(x,t)dx + \frac{2}{l}\int_0^l v^2(x,t)dx. \quad (6)$$

Both of them arise from obvious equalities

$$v(0,t) = \int_x^0 v_\xi(\xi,t)d\xi + v(x,t) \quad \text{and} \quad v(l,t) = \int_x^l v_\xi(\xi,t)d\xi + v(x,t).$$

Having applied (6), we get

$$\frac{1}{2}\int_0^\tau [v^2(0,t) + v^2(l,t)]dt \le 2l \int_0^\tau \int_0^l v_x^2(x,t)dxdt + \frac{2}{l}\int_0^\tau \int_0^l v^2(x,t)dxdt.$$

It follows from Theorem that there exist $c_0 > 0,\ a_0 > 0,\ a_1 > 0$ such that

$$\max_{Q_T} |c(x,t)| \le c_0,\ a(x,t) \ge a_0,\ \max_{Q_T} |a_t(x,t)| \le a_1.$$

Having used Cauchy inequality, we obtain $\left|\int_0^\tau \int_0^l cvv_t dxdt\right| \le \frac{c_0}{2}\int_0^\tau \int_0^l (v^2 + v_t^2)dxdt$. Hence we arrive at

$$\frac{1}{2}\int_0^l [v_t^2(x,\tau) + a_0 v_x^2(x,0) + v_{xt}^2(x,\tau)]dx \le (\frac{c_0}{2} + \frac{2}{l})\int_0^\tau \int_0^l v^2(x,t)dxdt +$$

$$\frac{c_0 + k_1 + k_2}{2}\int_0^\tau \int_0^l v_t^2(x,t)dxdt + (\frac{a_1}{2} + 2l)\int_0^\tau \int_0^l v_x^2(x,t)dxdt. \quad (7)$$

Note that from representation of v it follows an inequality

$$v^2(x,t) = \left(\int_\tau^t u(x,\eta)d\eta\right)^2 \le \tau \int_0^\tau u^2(x,t)dt = \tau \int_0^\tau v_t^2(x,t)dt,\ \tau \in [0,T].$$

Using it for estimation of the first term in the right side of (7), we get

$$\frac{1}{2}\int_0^l [v_t^2(x,\tau) + a_0 v_x^2(x,0) + v_{xt}^2(x,\tau)]dx \le C_1 \int_0^\tau \int_0^l v_t^2(x,t)dxdt + (\frac{a_1}{2} + 2l)\int_0^\tau \int_0^l v_x^2(x,t)dxdt,$$

where $C_1 = \max\{\frac{c_0\tau^2}{2},\ \frac{c_0+k_1+k_2}{2},\ \frac{2\tau^2}{l}\}$.

Introduce now a function $w(x,t) = \int\limits_0^t u_x(x,\eta)d\eta$. It is easy to see that it follows

$$v_x(x,t) = w(x,t) - w(x,\tau),\ v_x(x,0) = -w(x,\tau).$$

Taking into account these equalities, we get

$$\frac{1}{2}\int\limits_0^l [v_t^2(x,\tau) + a_0 w^2(x,\tau) + v_{xt}^2(x,\tau)]dx \leq C_2 \int\limits_0^\tau \int\limits_0^l (v_t^2 + w^2)dxdt + \tau(4l+a_1)\int\limits_0^l w^2(x,\tau)dx,$$

where $C_2 = \max\{C_1,\ 4l+a_1\}$. As τ is arbitrary we choose it in such a way that an inequality $\frac{a_0}{2} - \tau(4l+a_1) > 0$ holds. Let $\tau \in [0, \frac{a_0}{4(4l+a_1)}]$. Then $\frac{a_0}{2} - \tau(4l+a_1) \geq \frac{a_0}{4}$ and we can carry out $\tau(4l+a_1)\int\limits_0^l w^2(x,\tau)dx$ from the right side to the left side of the inequality. We now have

$$m_0\int\limits_0^l [v_t^2(x,\tau) + w^2(x,\tau) + v_{xt}^2(x,\tau)]dx \leq C_2 \int\limits_0^\tau \int\limits_0^l (v_t^2 + w^2)dxdt,$$

$m_0 = \min\{\frac{1}{2},\ \frac{a_0}{4}\}$. In particular,

$$\int\limits_0^l [v_t^2(x,\tau) + w^2(x,\tau)]dx \leq \frac{C_2}{m_0} \int\limits_0^\tau \int\limits_0^l (v_t^2 + w^2)dxdt,$$

and by virtue of Gronwall's inequality $v_t(x,\tau) = 0$, hence, $u(x,\tau) = 0\ \forall \tau \in [0, \frac{a_0}{4(4l+a_1)}]$. Following [7] we repeat these arguments for $\tau \in [\frac{a_0}{4(4l+a_1)}, \frac{a_0}{2(4l+a_1)}]$ and then continue this procedure. It follows that $u(x,\tau) = 0\ \forall \tau \in [0,T]$. It means that there exists at most one solution to the problem (1)—(3).

Existence. Let $w_k(x) \in C^2[0,l]$ be a basis in $W_2^1(0,l)$. We define the approximations

$$u^m(x,t) = \sum_{k=1}^m c_k(t)w_k(x), \tag{8}$$

where $c_k(t)$ are solutions to the Cauchy problem

$$\int\limits_0^l (u_{tt}^m w_j + a u_x^m w_j' + u_{xtt}^m w_j' + c u^m w_j)dx +$$
$$+ w_j(l)\int\limits_0^l K_2(x,t)u^m dx - w_j(0)\int\limits_0^l K_1(x,t)u^m dx = \int\limits_0^l f w_j dx, \tag{9}$$

$$c_k(0) = \alpha_k, \quad c_k'(0) = \beta_k, \tag{10}$$

where $\alpha_k,\ \beta_k$ — coefficients of the finite sums

$$\varphi^m(x)=\sum_{k=1}^{m}\alpha_k w_k(x),\quad \psi^m(x)=\sum_{k=1}^{m}\beta_k w_k(x),$$

approximating the functions φ and ψ as $m\to\infty$ in $W_2^1(0,l)$:

$$\sum_{k=1}^{m}\alpha_k w_k\to\varphi,\quad \sum_{k=1}^{m}\beta_k w_k\to\psi,\quad m\to\infty. \tag{11}$$

We write the Cauchy problem (9)— (10) as

$$\sum_{k=1}^{m}c''(t)A_{kj}+\sum_{k=1}^{m}c_k(t)B_{kj}(t)=f_j(t), \tag{12}$$

where

$$\begin{aligned}
&A_{kj}=(w_k,w_j)_{W_2^1(0,l)},\\
&B_{kj}(t)=\int_0^l[a(x,t)w_k'(x)w_j'(x)+c(x,t)w_k(x)w_j(x)]dx+\\
&\qquad+\int_0^l[K_2(x,t)w_k(x)w_j(l)-K_1(x,t)w_k(x)w_j(0)]dx,\\
&f_j(t)=\int_0^l f(x,t)w_j(x)dx.
\end{aligned}$$

Note that the matrix $||(w_k,w_j)_{W_2^1(0,l)}||$ is Gramian matrix as the functions w_k are linearly independent, hence the system (12) is normal. From theorem it follows that coefficients B_{kj} are bounded and $f_j\in L_1(0,T)$. Thus the Cauchy problem has a unique solution $c_k\in W_2^2(0,T)$ for every m and all approximations (8) are defined.

Next, we need a priori estimates to pass to the limit as $m\to\infty$.

Multiplying (9) by $c_j'(t)$, summing from $j=1$ to $j=m$ and integrating with respect to t from 0 to τ, we obtain

$$\begin{aligned}
&\int_0^\tau\int_0^l(u_{tt}^m u_t^m+au_x^m u_{xt}^m+u_{xtt}^m u_{xt}^m+cu^m u_t^m)dxdt+\int_0^\tau u_t^m(l,t)\int_0^l K_2(x,t)u^m(x,t)dxdt-\\
&\qquad-\int_0^\tau u_t^m(0,t)\int_0^l K_1(x,t)u^m(x,t)dxdt=\int_0^\tau\int_0^l f(x,t)u_t^m(x,t)dxdt.
\end{aligned} \tag{13}$$

Integrating by parts the first term in the left side of (13), we get

$$\int_0^l [(u_t^m(x,\tau))^2 + a(x,\tau)(u_x^m(x,\tau))^2 + (u_{xt}^m(x,\tau))^2]dx =$$

$$= \int_0^l [(u_t^m(x,0))^2 + a(x,0)(u_x^m(x,0))^2 + (u_{xt}^m(x,0))^2]dx + 2\int_0^\tau \int_0^l f u_t^m dxdt -$$

$$- 2\int_0^\tau \int_0^l c u^m u_t^m dxdt + 2\int_0^\tau u_t^m(0,t)\int_0^l K_1(x,t)u^m(x,t)dxdt -$$

$$- 2\int_0^\tau u_t^m(l,t)\int_0^l K_2(x,t)u^m(x,t)dxdt + \int_0^\tau \int_0^l a_t (u_x^m)^2 dxdt. \quad (14)$$

Consider the right side of (14) and focus our attention on terms generated by nonlocal conditions. By applying Cauchy-Bunyakovskii inequality, we get

$$\Big|2\int_0^\tau u_t^m(0,t)\int_0^l K_1(x,t)u^m(x,t)dxdt\Big| \le \int_0^\tau (u_t^m(0,t))^2 dt + k_1 \int_0^\tau \int_0^l (u^m(x,t))^2 dxdt,$$

$$\Big|2\int_0^\tau u_t^m(l,t)\int_0^l K_2(x,t)u^m(x,t)dxdt\Big| \le \int_0^\tau (u_t^m(l,t))^2 dt + k_2 \int_0^\tau \int_0^l (u^m(x,t))^2 dxdt,$$

where $k_i = \max\limits_{[0,T]} \int_0^l K_i^2(x,t)dx, \quad i = 1,2.$

In order to estimate the other terms of the right side we use the following inequalities

$$(u_t^m(0,t))^2 \le 2l\int_0^l (u_{xt}^m(x,t))^2 dx + \frac{2}{l}\int_0^l (u_t^m(x,t))^2 dx,$$

$$(u_t^m(l,t))^2 \le 2l\int_0^l (u_{xt}^m(x,t))^2 dx + \frac{2}{l}\int_0^l (u_t^m(x,t))^2 dx.$$

These inequalities follow from relations

$$u_t^m(0,t) = \int_x^0 u_{xt}^m(\xi,t)d\xi + u_t^m(x,t), \quad u_t^m(l,t) = \int_x^l u_{xt}^m(\xi,t)d\xi + u_t^m(x,t).$$

Then

$$\Big|\int_0^\tau (u_t^m(0,t))^2 dt\Big| + \Big|\int_0^\tau (u_t^m(l,t))^2 dt\Big| \le 4l\int_0^\tau \int_0^l (u_{xt}^m)^2 dxdt + \frac{4}{l}\int_0^\tau \int_0^l (u_t^m)^2 dxdt. \quad (15)$$

Now we apply Cauchy inequality to estimate the second and the third terms in the right side of (14) and get

$$2\Big|\int_0^\tau\int_0^l cu^m u_t^m dxdt\Big| \le c_0\int_0^\tau\int_0^l ((u^m)^2+(u_t^m)^2)dxdt,$$

$$2\Big|\int_0^\tau\int_0^l fu_t^m dxdt\Big| \le \int_0^\tau\int_0^l f^2dxdt+\int_0^\tau\int_0^l (u_t^m)^2dxdt.$$

With this result we can now obtain from (14) and (15)

$$\int_0^l [(u_t^m(x,\tau))^2+a(x,\tau)(u_x^m(x,\tau))^2+(u_{xt}^m(x,\tau))^2]dx \le$$

$$\le \int_0^l [(u_t^m(x,0))^2+a(x,0)(u_x^m(x,0))^2+(u_{xt}^m(x,0))^2]dx+\int_0^\tau\int_0^l f^2(x,t)dxdt+$$

$$+a_1\int_0^\tau\int_0^l (u_x^m)^2dxdt+c_0\int_0^\tau\int_0^l (u^m(x,t))^2dx+4l\int_0^\tau\int_0^l (u_{xt}^m(x,t))^2dxdt+$$

$$+(c_0+\frac{4}{l})\int_0^\tau\int_0^l (u_t^m(x,t))^2dxdt. \quad (16)$$

It easy to see that a relation

$$u^m(x,\tau)=\int_0^\tau u_t^m(x,t)dt+u^m(x,0)$$

implies the following inequality

$$\frac{1}{2}\int_0^l (u^m(x,\tau))^2dxdt \le \tau\int_0^\tau\int_0^l (u_t^m(x,t))^2dxdt+\int_0^l (u^m(x,0))^2dx.$$

Adding it to (16), we get:

$$m_0\int_0^l [(u^m(x,\tau))^2+(u_t^m(x,\tau))^2+(u_x^m(x,\tau))^2+(u_{xt}^m(x,\tau))^2]dx \le$$

$$\le M\int_0^\tau\int_0^l [(u^m(x,t))^2+(u_t^m(x,t))^2+(u_x^m(x,t))^2+(u_{xt}^m(x,t))^2]dxdt+$$

$$+N\int_0^l [(u^m(x,0))^2+(u_t^m(x,0))^2+(u_x^m(x,0))^2+(u_{xt}^m(x,0))^2]dx+\int_0^\tau\int_0^l f^2(x,t)dxdt,$$

where $M > 0, N > 0$ depend only on c_0, a_1, l, T. By Gronwall' lemma, we conclude that, for all $m \geq 1$,

$$||u^m||^2_{W(Q_T)} \leq C(||u^m(x,0)||^2_{W_2^1(0,l)} + ||u_t^m(x,0)||^2_{W_2^1(0,l)} + ||f||^2_{L_2(Q_T)}).$$

Taking into account (11) and hypotheses of the theorem we can state that $||u^m(x,0)||_{W_2^1(0,l)}$, $||u_t^m(x,0)||_{W_2^1(0,l)}$ are uniformly bounded with respect to m. Consequently,

$$||u^m||_{W(Q_T)} \leq P, \tag{17}$$

where $P > 0$ and does not depend on m.

Note that $W(Q_T)$ is Hilbert space. Therefore because of (17) we can extract from $\{u^m\}$ a subsequence convergent weakly in $W(Q_T)$ and uniformly with respect to $t \in [0,T]$ in the norm of $L_2(0,l)$ to $u \in W(Q_T)$.

We need only to show that this limit function is a required generalized solution.

Initial data $u(x,0) = \varphi(x)$ is fulfilled as $u^m(x,t) \to u(x,t)$ in $L_2(0,l)$ uniformly for every $t \in [0,T]$ and $u^m(x,0) \to \varphi$ in $L_2(0,l)$ (see (11)). In order to show that (4) is valid we multiply (9) by $d_j \in C^1[0,T]$, $d_j(T) = 0$, sum from $j = 1$ to $j = m$, then integrate with respect to t from 0 to T.

Denote $\eta(x,t) = \sum\limits_{j=1}^{m} d_j(t)w_j(x)$. After integrating by parts the terms containing u_{tt}^m and u_{xtt}^m we get

$$\int_0^T \int_\Omega (-u_t^m \eta_t - u_{xt}^m \eta_{xt} + a u_x^m \eta_x + c u^m \eta)\, dxdt - \int_0^l u_t^m(x,0)\eta(x,0)dx - \int_0^l u_{xt}^m(x,0)\eta_x(x,0)dx +$$
$$+ \int_0^T \eta(l,t) \int_0^l K_2(x,t)u^m(x,t)dxdt - \int_0^T \eta(0,t) \int_0^l K_1(x,t)u^m(x,t)dxdt = \int_0^T \int_0^l f\eta\, dxdt. \tag{18}$$

Taking into account convergences proved above one can pass to the limit in (18) as $m \to \infty$ for any fixed η. Denote the set of functions $\eta = \sum\limits_{j=1}^{m} d_j(t)w_j(x)$ by $\mathcal{N}_m$. As $\bigcup\limits_{m=1}^{\infty} \mathcal{N}_m$ is dense in $\hat{W}$ it follows that the limit relation is fulfilled for every function $v \in \hat{W}(Q_T)$, hence, u is the solution to the problem (1)—(3). □

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