UDC 539.3

Wang Kai-xing[1], Pan Yi-shan[1], N. Dergachova[2]

STEADY-STATE RESPONSE AND RESONANCE CONDITION OF BLOCK ROCK MASS ON EXTERNAL PERIODIC EXCITATION

Based on the discontinuous and self-stress rock mass in depth the theory of deep rock mass as of a complex hierarchy of block structure is proposed. The block structure of rock mass steady-state response on external periodic excitation is studied. We get resonance equation and resonance condition of the block rock mass structure on external periodic excitation. The effect of local mass and stress state between adjacent rock blocks in block rock mass structure to the steady-state displacement of rock blocks are analyzed.

Introduction. Deep rock mass has a complicated geological environment of high ground stress, high temperature and high osmotic pressure. The rock mass structure in deep is critical for studying of deep rock mass dynamic response. Russia scholar M. A. Sadovskii [16] put forward the theory of deep rock mass as a complex hierarchy of block structure and theory was proved by studying of geomechanics field. Rock mass in deep have a block structure with different hierarchy from crystal to rock mass, and between the adjacent blocks with weak mechanical characteristics, which have segmentation as block structure. The block structure has different self-similar hierarchy scale. The j-hierarchy geological rock mass has the scale Δ_j:

$$\Delta_j = (\sqrt{2})^{-j} \Delta_0, \qquad j = 0, 1, 2, \ldots, \tag{1}$$

where $\Delta_0 = 2.5 \times 10^6$ m. M. V. Kurlenya [9] found the phenomenon of rock mass signal changed alternately when rock mass have explosion effect. As a result based on the theory of elastic-plastic stress wave of continuum mechanics can not given a reasonable interpretation. M. V. Kurlenya conjecture, that there is a new wave called pendulum-type wave [11]. The production of pendulum-type wave is closely related to geological structure. M. A. Sadovskii [16] put forward a new viewpoint and it makes dynamic study of rock mass based on discontinuous and self-stress block rock mass structure. Today researches show that pendulum-type wave propagation in the discontinuous and self-stress block rock mass structure is closely related with rock mass dynamic response in deep [10, 12–14]. Last years theoretical and experimental study of pendulum-type wave achieved some progress [1–6, 8, 15, 17].

This paper studies the steady-state response of rock blocks on external periodic excitation. We get resonance condition of the block rock mass structure. Steady-state response is compared on different external excitation frequencies. The effect of block rock mass structure parameters including mass and visco-elasticity between adjacent blocks to the steady-state displacement at the same external periodic excitation are also compared.

1. Problem Statement and Block Rock Mass Structure Resonance Analysis. Deep rock mass is a system as a complex hierarchy of block structure in which rock blocks are nested in one another and mutually linked by intermediate layers composed of weaker and fractured rocks. A model with partings and parallel arranged elastic and damping elements is proposed in paper [2] as Fig. 1. In the form of chains of blocks with friable layers between them showed that wave propagation in such environment is sufficiently described by an approximation that the blocks are non-deformable bodies.

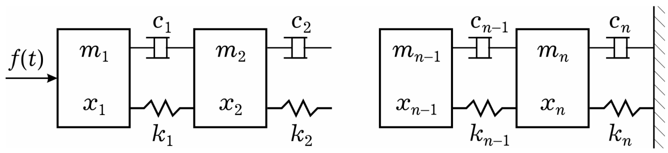


Fig. 1. Model of block rock mass structure with visco-elastically linked.

Dynamic equations of block rock mass structure at external periodic excitation expressed as matrix form

$$M \times \ddot{x}(t) + C \times \dot{x}(t) + K \times x(t) = F(t)\,, \tag{2}$$

where

$$M = \begin{bmatrix} m_1 & & & \\ & m_2 & & \\ & & \ddots & \\ & & & m_n \end{bmatrix},$$

$$C = \begin{bmatrix} c_1 & -c_1 & & & & \\ -c_1 & (c_1 + c_2) & -c_2 & & & \\ & \ddots & \ddots & \ddots & & \\ & & -c_{i-1} & (c_{i-1} + c_i) & -c_i & \\ & & & \ddots & \ddots & \ddots \quad -c_{n-1} \\ & & & & -c_{n-1} & (c_{n-1} + c_n) \end{bmatrix},$$

$$K = \begin{bmatrix} k_1 & -k_1 & & & & \\ -k_1 & (k_1 + k_2) & -k_2 & & & \\ & \ddots & \ddots & \ddots & & \\ & & -k_{i-1} & (k_{i-1} + k_i) & -k_i & \\ & & & \ddots & \ddots & \ddots \quad -k_{n-1} \\ & & & & -k_{n-1} & (k_{n-1} + k_n) \end{bmatrix},$$

$x = [x_1, \cdots, x_n]$ is the displacement of each block, $F(t) = [f(t), 0, \ldots, 0]$ and $f(t)$ is the function of external periodic excitation.

Reduced order of equations (2):

$$\begin{bmatrix} C & M \\ M & 0 \end{bmatrix} \begin{bmatrix} \dot{x} \\ \ddot{x} \end{bmatrix} + \begin{bmatrix} K & 0 \\ 0 & -M \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \end{bmatrix} = \begin{bmatrix} F(t) \\ 0 \end{bmatrix}. \tag{3}$$

Let be

$$y(t) = \begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix}. \tag{4}$$

As a result, formula (3) has the form

$$A\dot{y}(t) + By(t) = \tilde{f}(t)\,, \tag{5}$$

where

$$A = \begin{bmatrix} C & M \\ M & 0 \end{bmatrix}, \qquad B = \begin{bmatrix} K & 0 \\ 0 & -M \end{bmatrix}, \qquad \tilde{f}(t) = \begin{bmatrix} F(t) \\ 0 \end{bmatrix}.$$

Let be $y = \Phi q$, in state space pendulum-type wave equations (5) can be decoupled into first order differential equations

$$\dot{q} - \lambda_j q = \frac{(\Phi^\top \tilde{f})_{j,1}}{a_{jj}}, \qquad j = 1, 2, \ldots, 2n\,, \tag{6}$$

where λ_j and φ_j are generalized eigenvalue and generalized eigenvector of $B^{-1}A\varphi = \frac{1}{\lambda}\varphi$, Φ is a matrix comprised of generalized eigenvector φ_j, $j = 1,2,\ldots,2n$, $a = \Phi^{\top}A\Phi = \mathrm{diag}(a_1, a_2, \ldots, a_{2n})$.

As a result, solution of equation (6)

$$q_j(t) = q_j(0)e^{\lambda_j t} + w_j\,, \tag{7}$$

where

$$w_j = e^{\lambda_j t}\int_0^t \frac{(\Phi^{\top}\tilde{f})_{j,1}}{a_{jj}} e^{-\lambda_j t}\,dt, \qquad q(0) = a^{-1}\Phi^{\top}Ay(0)\,,$$

$y(0)$ is the initial condition. Matrix form of (7)

$$q = dq(0) + W\,, \tag{8}$$

where

$$d = \mathrm{diag}(e^{\lambda_1 t}, e^{\lambda_2 t}, \ldots, e^{\lambda_{2n} t}), \qquad W = [w_1, w_2, \ldots, w_{2n}]^{\top}.$$

As a result,

$$y(t) = \Phi(dq(0) + W)\,. \tag{9}$$

If $y(0) = 0$, without considering transient solution, the steady-state solution of (9) with external periodic excitation $f(t) = p\sin(\omega t)$ as

$$y(t) = \Phi W \qquad \text{and} \qquad y_i(t) = \sum_{j=1}^{2n}\Phi_{ij}\cdot w_j\,, \tag{10}$$

and

$$w_j = e^{\lambda_j t}\int_0^t \frac{\Phi_{j,1}\cdot p\cdot\sin(\omega t)}{a_{jj}} e^{-\lambda_j t}\,dt =$$

$$= -\frac{A_j\cdot(\lambda_j\sin(\omega t) + \omega\cos(\omega t)) - A_j\cdot\omega\cdot e^{\lambda_j t}}{\lambda_j^2 + \omega^2}\,, \tag{11}$$

where $A_j = \frac{p\cdot\Phi_{j,1}}{a_{jj}}$, $\lambda_j \in C$, $j = 1,2,\ldots,2n$, is complex frequency of the j-order, ω is the frequency of external periodic excitation. As known from formula (9), block rock mass structure resonance only caused by the steady-state response at external periodic excitation. As known from formula (11), the resonance occurs when λ_j is a purely imaginary number and $\mathrm{Im}(\lambda_j) = \omega$, in this case, $\lambda_j^2 + \omega^2 = 0$ and block rock mass structure resonance . We call equation (11) as resonance equation of block rock mass structure. There are n resonance frequencies in the block rock mass structure with n rock blocks. The displacement of block is the linear combination of w_j and arbitrary term of $\sum_{j=1}^{2n}\Phi_{ij}\cdot w_j$ resonance can cause the block rock mass structure resonance. In rock engineering, when imaginary part of λ_j much more than real part and external periodic excitation frequency closed to imaginary part of λ_j can be seen as quasi-resonance.

Example.

External periodic excitation $f(t) = \sin(\omega t)$ and the block rock mass structure composed of ten blocks. The mass of each block $m_j = 10\,\text{kg}$, $j = 1,\ldots,10$, viscosity coefficient between blocks $c_j = 20\,\text{kg/s}$, $j = 1,\ldots,10$; elastic coefficient between blocks $k_j = 1\times 10^5\,\text{kg/s}$, $j = 1,\ldots,10$. The first term of steady-state displacement response in x_5 is $\Phi_{5,1}w_1$ and its response is compared on different external excitation frequency as Fig. 2.

Complex frequencies of block rock mass structure:

$\omega_1 = -3.91 \pm 197.73\,i$, $\omega_2 = 3.65 \pm 191.08\,i$, $\omega_3 = -3.25 \pm 180.16\,i$,

$\omega_4 = -2.73 \pm 165.23\,i$, $\omega_5 = -2.15 \pm 146.59\,i$, $\omega_6 = -1.55 \pm 124.69\,i$,

$\omega_7 = -1.00 \pm 99.99\,i$, $\omega_8 = -0.53 \pm 73.07\,i$, $\omega_9 = -0.20 \pm 44.50\,i$,

$\omega_{10} = -0.02 \pm 14.95\,i$.

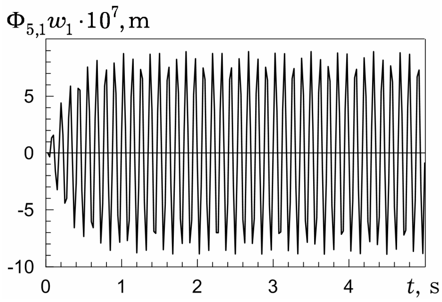
Band center and bandwidth of the steady-state displacement in frequency domain as formula

$$\langle\omega\rangle = \frac{1}{E}\int \omega \cdot \left|F_j(\omega)\right|^2 d\omega \quad \text{and} \quad \sigma_\omega = \left[\int (\omega - \langle\omega\rangle)^2 \left|F_j(\omega)\right|^2 d\omega\right]^{1/2}, \qquad (12)$$

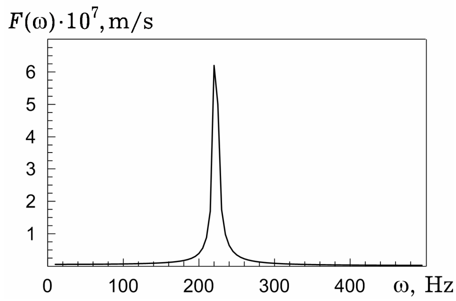
where

$$E = \int \left|F_j(\omega)\right|^2 d\omega, \qquad F_j(\omega) = \int_{-\infty}^{+\infty} \ddot{x}_j(t) e^{-i\omega t}\, dt.$$

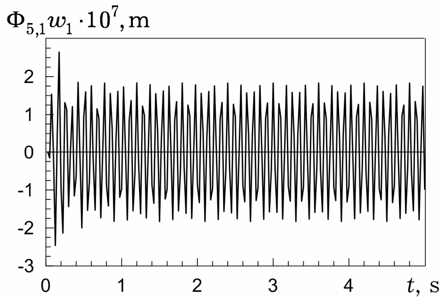
As a result, the response in frequency domain concentrate in range of $\langle\omega\rangle \pm \sigma_\omega$.



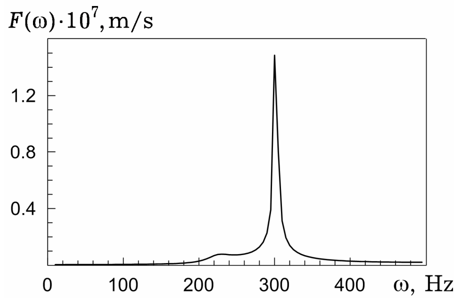
(***a***) $\Phi_{5,1}w_1$ response when $\omega = 198$



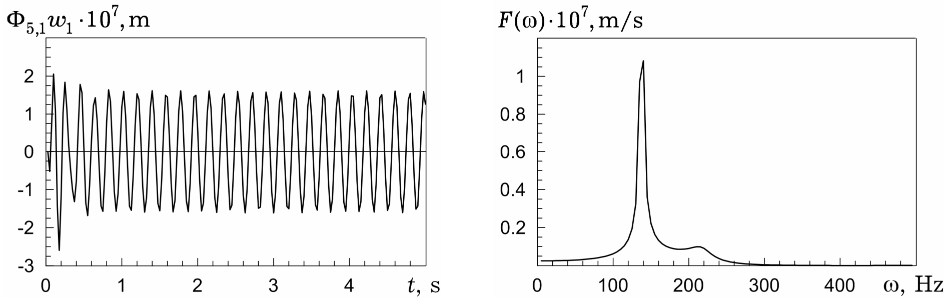
(***b***) frequency response corresponding (***a***)



(***c***) $\Phi_{5,1}w_1$ response when $\omega = 178$



(***d***) frequency response corresponding (***c***)



(*e*) $\Phi_{5,1}w_1$ response when $\omega = 218$ (*f*) frequency response corresponding (*e*)

Fig. 2. Steady-state displacement response of $\Phi_{5,1}w_1$ in time-domain and frequency-domain.

Imaginary part of the first-order complex frequency is 197.73HZ. When external excitation frequency $\omega = 198$ closed to imaginary part of the first-order complex frequency. The amplitude of $\Phi_{5,1}w_1$ increased about 5 times compared with $\omega = 178$ and $\omega = 218$. In this example doesn't appear resonance phenomenon because of λ_1 is not a purely imaginary number.

From the formula (12) we get $\Phi_{5,1}w_1$ frequency response, the frequency band center $\langle\omega\rangle = 212.07$ and bandwidth $\sigma_\omega = 9.68$ when $\omega = 198$; band center $\langle\omega\rangle = 290.63$ and bandwidth $\sigma_\omega = 15.23$, when $\omega = 178$; band center $\langle\omega\rangle = 134.88$ and bandwidth $\sigma_\omega = 16.22$ when $\omega = 218$. Band center of frequency response higher when $\omega = 178$ than $\omega = 198$, but when $\omega = 218$ the band center is lower compared with $\omega = 198$. Frequency bandwidth of $\Phi_{5,1}w_1$ on another two external excitation frequencies are more than $\omega = 198$.

2. Block rock mass structure parameters sensitivity to steady-state response

$$\frac{\partial w_i}{\partial m_k} = \frac{\partial w_i}{\partial \lambda_i}\cdot\frac{\partial \lambda_i}{\partial m_k} + \frac{\partial w_i}{\partial \varphi_{i1}}\cdot\frac{\partial \varphi_{i1}}{\partial m_k}, \tag{13-a}$$

$$\frac{\partial w_i}{\partial c_{k\ell}} = \frac{\partial w_i}{\partial \lambda_i}\cdot\frac{\partial \lambda_i}{\partial c_{k\ell}} + \frac{\partial w_i}{\partial \varphi_{i1}}\cdot\frac{\partial \varphi_{i1}}{\partial c_{k\ell}}, \tag{13-b}$$

$$\frac{\partial w_i}{\partial k_{k\ell}} = \frac{\partial w_i}{\partial \lambda_i}\cdot\frac{\partial \lambda_i}{\partial k_{k\ell}} + \frac{\partial w_i}{\partial \varphi_{i1}}\cdot\frac{\partial \varphi_{i1}}{\partial k_{k\ell}}, \tag{13-c}$$

where m_k is the mass of block k, $c_{k\ell}$ and $k_{k\ell}$ are viscosity coefficient and elastic coefficient between adjacent blocks k and ℓ:

$$w_i = -\frac{A_i\cdot(\lambda_i \sin\omega t + \omega\cos\omega t) - A_i\cdot\omega\cdot e^{\lambda_i t}}{\lambda_i^2 + \omega^2},$$

therefore

$$\frac{\partial w_i}{\partial \lambda_i} = -\frac{A_i(\sin\omega t - \omega\lambda_i e^{\lambda_i t})(\lambda_i^2+\omega^2) - 2A_i\lambda_i(\lambda_i\sin\omega t + \omega\cos\omega t - \omega e^{\lambda_i t})}{(\lambda_i^2+\omega^2)^2} \tag{14-a}$$

$$\frac{\partial w_i}{\partial \varphi_{i1}} = -\frac{p(\lambda_i \sin\omega t + \omega\cos\omega t) - p\omega e^{\lambda_i t}}{a_{ii}(\lambda_i^2+\omega^2)}. \tag{14-b}$$

Paper [7] given sensitivity analysis of λ_r, φ_{ir} to local mass and visco-elasticity:

$$\frac{\partial \lambda_r}{\partial m_k} = -\lambda_r^2 \frac{\varphi_{kr}^2}{a_r}, \tag{15-a}$$

$$\frac{\partial \lambda_r}{\partial c_{k\ell}} = -\lambda_r \frac{(\varphi_{kr} - \varphi_{\ell r})^2}{a_r}, \tag{15-b}$$

$$\frac{\partial \lambda_r}{\partial k_{k\ell}} = -\frac{(\varphi_{kr} - \varphi_{\ell r})^2}{a_r}, \tag{15-c}$$

$$\frac{\partial \varphi_{ir}}{\partial m_k} = -\lambda_r \frac{\varphi_{kr}^2}{a_r} \varphi_{ir} + \varphi_{kr} \sum_{\substack{j=1,\\ j\neq r}}^{2N} \frac{\lambda_r^2}{\lambda_j - \lambda_r} \frac{\varphi_{kj}\varphi_{ij}}{a_j}, \tag{15-d}$$

$$\frac{\partial \varphi_{ir}}{\partial c_{k\ell}} = -\frac{1}{2} \frac{(\varphi_{kr} - \varphi_{\ell r})^2}{a_r} \varphi_{ir} + (\varphi_{kr} - \varphi_{\ell r}) \sum_{\substack{j=1,\\ j\neq r}}^{2N} \frac{\lambda_r}{\lambda_j - \lambda_r} \frac{(\varphi_{kj} - \varphi_{\ell j})\varphi_{ij}}{a_j}, \tag{15-e}$$

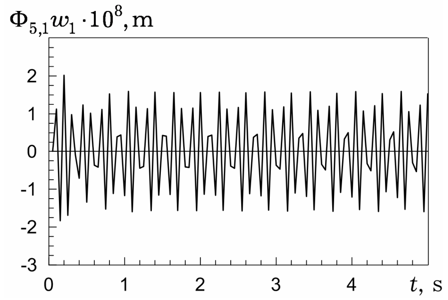
$$\frac{\partial \varphi_{ir}}{\partial k_{k\ell}} = (\varphi_{kr} - \varphi_{\ell r}) \sum_{\substack{j=1,\\ j\neq r}}^{2N} \frac{1}{\lambda_j - \lambda_r} \frac{(\varphi_{kj} - \varphi_{\ell j})\varphi_{ij}}{a_j}. \tag{15-f}$$

Inserting formula (14-*a*)–(14-*b*) and formula (15-*a*)–(15-*f*) to formula (13-*a*)–(13-*c*) we can get the effect of local mass and visco-elasticity to steady-state displacement response.

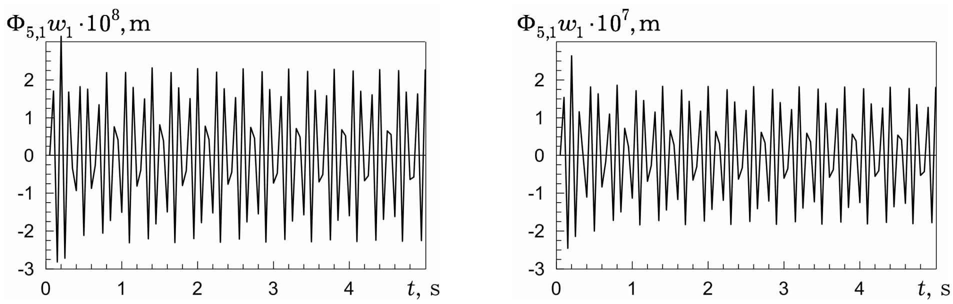
Example.

The effect of local parameters including mass of block and visco-elasticity between adjacent blocks to the block steady-state displacement response on certain external periodic excitation $f(t) = \sin(178 \times t)$ is compared. We are also analysis $\Phi_{5,1}w_1$ when change local parameters $m_5 = 20$ kg, $c_4 = 40$ kg/s; $k_4 = 0.5 \cdot 10^5$ kg/s^2 respectively and other parameters invariant. We can compare the results with upper example when $\omega = 178$.

The variation of $\Phi_{5,1}w_1$ is as Fig. 3. Amplitude of $\Phi_{5,1}w_1$ decreased when local mass m_5 increase to two times or elasticity-coefficient k_4 decrease to 1/2. Amplitude of $\Phi_{5,1}w_1$ no changed, when local viscous-coefficient c_4 increase to two times. Local mass and elasticity coefficient have a great influence to $\Phi_{5,1}w_1$, compared with viscous coefficient.



(*a*) change mass of fifth block $m_5 = 20$ kg



(***b***) change elastic coefficient $k_4 = 0.5 \cdot 10^5\ \mathrm{kg/s^2}$

(***c***) change elastic coefficient $c_4 = 40\ \mathrm{kg/s}$

Fig. 3. Response of $\Phi_{5,1} w_1$ when changing local parameters in block rock mass structure on certain external excitation frequency $\omega = 178$.

Conclusion. Through analysis the rock bock steady-state response on external periodic excitation, we know that block rock mass structure quasi-resonance when frequency of external excitation close to imaginary part of the rock system complex frequency λ_j, and when λ_j is a purely imaginary number there will be resonance. In rock engineering through testing the block rock mass structure parameters, we can calculate complex frequency of the rock mass system. As a result, frequency of external excitation should avoid these frequencies value in theory.

When external excitation frequency approximates the resonance frequency from the lower, the frequency band center of the steady-state response exceed to the band center caused by resonance frequency. But from the higher, the frequency band center is below. When change local parameters in block rock mass structure, complex frequency of rock block system corresponding changed. Local mass and elasticity coefficient have a great influence on steady-state displacement response compared with viscous coefficient .In rock mechanics and engineering when frequency of external excitation difficult to change, we can change local parameters to ensure the stability of the rock mass structure.

The study was conducted with financial support from the China Foundation for Basic Research, Project No. 2010CB226803.

1. *Aleksandrova N. I., Ayzenberg-Stepanenko M. V., Sher E. N.* Modeling the elastic wave propagation in a block medium under the impulse loading // J. Min. Sci. – 2009. – **45**, No. 5. – P. 427–437.
2. *Aleksandrova N. I., Chernikov A. G., Sher E. N.* Experimental investigation into the one-dimensional calculated model of wave propagation in block medium // J. Min. Sci. – 2005. – **41**, No. 3. – P. 232–239.
3. *Aleksandrova N. I., Chernikov A. G., Sher E. N* On attenuation of pendulum-type waves in a block rock mass // J. Min. Sci. – 2006. – **42**, No. 5. – P. 556–564.
4. *Aleksandrova N. I., Sher E. N.* Modeling of wave propagation in block media // J. Min. Sci. – 2004. – **40**, No. 6. – P. 579–587.
5. *Aleksandrova N. I., Sher E. N.* Wave propagation in the 2d periodical model of a block-structured medium. Part I: characteristics of waves under impulsive impact // J. Min. Sci. – 2010. – **46**, No. 6. – P. 639–649.
6. *Aleksandrova N. I., Sher E. N., Chernikov A. G.* Effect of viscosity of partings in block-hierarchical media on propagation of low-frequency pendulum waves // J. Min. Sci. – 2008. – **44**, No. 3. – P. 225–234.
7. *Allemang R. J., Brown D. L, Soni M. L.* Experimental modal analysis and dynamic component synthesis. – Final Technical Report AFWAL-TR-87-3069. – Vol. IV: System modeling techniques. – 153 p.
8. *Bagaev S. N., Oparin V. N., Orlov V. A., Panov S. V., Parushkin M. D.* Pendulum waves and their singling out in the laser deformograph records of the large earthquakes // J. Min. Sci. – 2010. – **46**, No. 3. – P. 217–224.

9. *Kurlenya M. V., Adushkin V. V., Garnov V. V., Oparin V. N., Revuzhenko A. F., Spivak A. A.* Variable-sign reaction of rocks on a dynamic action // Dokl. Akad. Nauk SSSR. − 1992. − **323**, No. 2. − P. 263−265.
10. *Kurlenya M. V., Oparin V. N.* Problems of nonlinear geomechanics. Part 1 // J. Min. Sci. − 1999. − **35**, No. 3. − P. 216−230.
11. *Kurlenya M. V., Oparin V. N., Vostrikov V. I.* Formation of elastic wave packets by the impulse agitation of block masses. The wave of pendulum-type v_{μ} // Dokl. Akad. Nauk SSSR. − 1993. − **333**, No. 4. − P. 515−521.
12. *Kurlenya M. V., Oparin V. N., Vostrikov V. I.* Pendulum-type waves. Part I: State of the problem and measuring instrument and computer complexes // J. Min. Sci. −1996. − **32**, No. 3. − P. 159−163.
13. *Kurlenya M. V., Oparin V. N., Vostrikov V. I.* Pendulum-type waves. Part II: Experimental methods and main results of physical modeling // J. Min. Sci. − 1996. − **32**, No. 4. − P. 245−273.
14. *Kurlenya M. V., Oparin V. N., Vostrikov V. I.* Pendulum waves. Part III: Data of on-site observations // J. Min. Sci. − 1996. − **32**, No. 5. − P. 341−361.
15. *Oparin V. N., Simonov B. F.* Nonlinear deformation-wave processes in the vibrational oil geotechnologies // J. Min. Sci. − 2010. − **46**, No. 2. − P. 95−112.
16. *Sadovskii M. A.* Natural lumpiness of a rock // Dokl. Akad. Nauk SSSR. − 1979. − **247**, No. 4. − P. 829−831.
17. *Sher E. N., Aleksandrova N. I., Ayzenberg-Stepanenko M. V., Chernikov A. G.* Influence of the block-hierarchical structure of rocks on the peculiarities of seismic wave propagation // J. Min. Sci. −2007. − **43**, No. 6. − P. 585−591.

РЕАКЦІЯ СТАЦІОНАРНОГО СТАНУ ТА УМОВА РЕЗОНАНСУ БЛОКУ ГІРСЬКОЇ МАСИ ПРИ ЗОВНІШНЬОМУ ПЕРІОДИЧНОМУ ЗБУРЕННІ

На основі моделі розривної і самонапруженої у глибині гірської маси розгляається теорія глибокої гірської маси як складної системи блочної структури. Вивчатья стаціонарний стан структури блоку гірської маси при зовнішніх періодичних збуреннях. Отримано рівняння резонансу структури блоку гірської маси і умови резонансу при зовнішньому періодичному збуренні. Аналізується вплив локальних мас і локального напруженого стану між сусідніми блоками породи в структурі блоку гірської маси як реакція на стаціонарне зміщення гірських порід.

РЕАКЦИЯ СТАЦИОНАРНОГО СОСТОЯНИЯ И УСЛОВИЕ РЕЗОНАНСА БЛОКА ГОРНОЙ МАССЫ ПРИ ВНЕШНЕМ ПЕРИОДИЧЕСКОМ ВОЗБУЖДЕНИИ

На основе модели разрывной и самонапряженной в глубине горной массы рассматривается теория глубокой горной массы как сложной системы блочной структуры. Изучается стационарное состояние структуры блока горной массы при внешних периодических возбуждениях. Получены уравнение резонанса структуры блока горной массы и условия резонанса при внешнем периодическом возбуждении. Анализируется влияние локальных масс и локального напряженного состояния между соседними блоками породы в структуре блока горной массы как реакция на стационарное смещение горных пород.

[1] School of Mechanics and Engineering,
Liaoning Techn. Univ., Liaoning, China,
[2] Donetsk Nat. Univ., Donetsk, Ukraine

Received
13.10.12