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PROBLEM WITH INTEGRAL CONDITIONS FOR DIFFERENTIAL-OPERATOR EQUATION

We propose a method of solving the problem with inhomogeneous integral conditions for homogeneous differential-operator equation with abstract operator in a linear space H. For right-hand sides of the integral conditions which belong to the special subspace $L \subseteq H$, in which the vectors are represented in the form of Stieltjes integrals with respect to certain measures, the solution of the problem is represented in the form of Stieltjes integrals with respect to the same measures. We give the example of applying the method to solving the ill-posed problem for the second order partial differential equation in time variable (in which the integral conditions are given) and, generally, an infinite order partial differential equation in spatial variable.

Introduction. In recent years, problems with integral conditions for partial differential equations and differential-operator equations have been widely investigated and found a lot of applications in important practical problems. Integral conditions are used, in particular, in models of heat distribution and moisture transfer, in demographical models, in problems of mathematical biology and optimal control in technology etc. [7, 11–13, 16].

Determining the conditions of correct solvability of the problems with integral conditions for differential and differential-operator equations has been a subject of wide range of papers (see, e. g., [1, 2, 6, 8–10, 15] and the bibliography there).

The present work is a continuation of the investigations [3, 5]. Here we propose a method of solving a problem with inhomogeneous integral conditions for homogeneous differential-operator equation with abstract operator in a linear space. The solution of the problem is represented in the form of Stieltjes integral with respect to certain measures.

1. Statement of the problem. Let A be a given linear operator acting in the linear space H and, for this operator, arbitrary powers A^n, $n = 2, 3, \ldots$, be also defined in H.

We consider the problem

$$\left[\frac{d}{dt} - a(A)\right]^2 U(t) = 0, \qquad t \in (0,h), \tag{1}$$

$$\int_0^h U(t)\,dt = \varphi_1, \qquad \int_0^h tU(t)\,dt = \varphi_2, \tag{2}$$

where $\varphi_1, \varphi_2 \in H$, $U : (0,h) \to H$ is an unknown vector-function, $a(A)$ is an abstract operator with entire symbol $a(\lambda) = \sum_{k=1}^{\infty} a_k \lambda^k \ne \text{const}$.

We assume that $a(A)U = \sum_{k=1}^{\infty} a_k A^k U$. Besides, we assume that operator A commutes with $\frac{d}{dt}$, then the operators $a(A)$ and $\frac{d}{dt}$ also commute, hence

$$\left[\frac{d}{dt} - a(A)\right]^2 U = \frac{d^2 U}{dt^2} - 2\frac{d}{dt}(a(A)U) + a^2(A)U .$$

Along the lines of equation (1), replacing A by $\lambda \in \mathbb{C}$ we write down the ordinary differential equation

$$\left[\frac{d}{dt}-a(\lambda)\right]^2 T = 0\,. \tag{3}$$

The general solution of equation (3) (considering its dependence on the parameter λ) looks as follows:

$$T(t,\lambda) = C_1(\lambda)e^{a(\lambda)t} + C_2(\lambda)te^{a(\lambda)t}, \tag{4}$$

where $C_1(\lambda)$ and $C_2(\lambda)$ are arbitrary functions of parameter λ.

We shall find the solutions $\hat{T}_1(t,\lambda)$, $\hat{T}_2(t,\lambda)$ of equation (3), which satisfy the conditions

$$\int_0^h \hat{T}_1(t,\lambda)dt = 1, \qquad \int_0^h t\hat{T}_1(t,\lambda)dt = 0\,, \tag{5}$$

$$\int_0^h \hat{T}_2(t,\lambda)dt = 0, \qquad \int_0^h t\hat{T}_2(t,\lambda)dt = 1\,. \tag{6}$$

We search for $\hat{T}_1(t,\lambda)$ in the form (4), where $C_1(\lambda)$ and $C_2(\lambda)$ are unknown functions of parameter λ. Satisfying conditions (5), we obtain the system of equations

$$C_1(\lambda)I_0(\lambda) + C_2(\lambda)I_1(\lambda) = 1\,,$$

$$C_1(\lambda)I_1(\lambda) + C_2(\lambda)I_2(\lambda) = 0\,, \tag{7}$$

where

$$I_0(\lambda) = \int_0^h e^{a(\lambda)t}\,dt = \frac{e^{a(\lambda)h}-1}{a(\lambda)}\,,$$

$$I_1(\lambda) = \int_0^h te^{a(\lambda)t}\,dt = \frac{ha(\lambda)I_0(\lambda) + h - I_0(\lambda)}{a(\lambda)}\,,$$

$$I_2(\lambda) = \int_0^h t^2e^{a(\lambda)t}\,dt = \frac{a^2(\lambda)h^2I_0(\lambda) + a(\lambda)h^2 - 2ha(\lambda)I_0(\lambda) - 2h + 2I_0(\lambda)}{a(\lambda)}.$$

Note that if $\lambda_0 \in M_0$, where $M_0 = \{\lambda \in \mathbb{C} : a(\lambda) = 0\}$, then $I_0(\lambda_0) = h$, $I_1(\lambda_0) = \frac{h^2}{2}$, $I_2(\lambda) = \frac{h^3}{3}$.

The main determinant of system (7) is

$$\Delta(\lambda) = \begin{vmatrix} I_0(\lambda) & I_1(\lambda) \\ I_1(\lambda) & I_2(\lambda) \end{vmatrix} = I_0(\lambda)I_2(\lambda) - I_1^2(\lambda) = \frac{I_0^2(\lambda) - a(\lambda)h^2I_0(\lambda) - h^2}{a^2(\lambda)}.$$

Taking into account the assumption for $I_0(\lambda)$, we find

$$\Delta(\lambda) = \frac{(e^{a(\lambda)h}-1)^2 - h^2a^2(\lambda)e^{a(\lambda)h}}{a^4(\lambda)}, \tag{8}$$

and besides, if $\lambda_0 \in M_0$, then

$$\Delta(\lambda_0) = \frac{h^4}{12}.$$

We denote

$$M = \{\lambda \in \mathbb{C} : \Delta(\lambda) = 0\}. \tag{9}$$

Now calculate determinants:

$$\Delta_{11}(\lambda) = \begin{vmatrix} 1 & I_1(\lambda) \\ 0 & I_2(\lambda) \end{vmatrix} = I_2(\lambda),$$

$$\Delta_{12}(\lambda) = \begin{vmatrix} I_0(\lambda) & 1 \\ I_1(\lambda) & 0 \end{vmatrix} = -I_1(\lambda).$$

For $\lambda \in \mathbb{C} \setminus M$, we find

$$\hat{T}_1(t,\lambda) = \frac{\Delta_{11}(\lambda) + t\Delta_{12}(\lambda)}{\Delta(\lambda)} e^{a(\lambda)t} = \frac{I_2(\lambda) - tI_1(\lambda)}{\Delta(\lambda)} e^{a(\lambda)t},$$

that yields

$$\hat{T}_1(t,\lambda) = [(a^2(\lambda)h^2 - 2a(\lambda)h + 2)e^{a(\lambda)h} - 2 - a(\lambda)te^{a(\lambda)h}(a(\lambda)h - 1) - a(\lambda)] \times$$
$$\times \frac{e^{a(\lambda)t}}{a^3(\lambda)\Delta(\lambda)}. \tag{10}$$

In the same way, we search for $\hat{T}_2(t,\lambda)$ in the form (4), satisfying conditions (6):

$$\hat{T}_2(t,\lambda) = \frac{\Delta_{21}(\lambda) + t\Delta_{22}(\lambda)}{\Delta(\lambda)} e^{a(\lambda)t} = \frac{-I_1(\lambda) + tI_0(\lambda)}{\Delta(\lambda)} e^{a(\lambda)t},$$

that implies

$$\hat{T}_2(t,\lambda) = \frac{-(a(\lambda)h - 1)e^{a(\lambda)h} - 1 + a(\lambda)t(e^{a(\lambda)h} - 1)}{a^2(\lambda)\Delta(\lambda)} e^{a(\lambda)t}. \tag{11}$$

We note once more time that if $\lambda_0 \in M_0$, then

$$\hat{T}_1(t,\lambda_0) = \frac{2(2h - 3t)}{h^2}, \qquad \hat{T}_2(t,\lambda_0) = \frac{6(2t - h)}{h^3}.$$

Lemma 1. *If* $P = \{\nu \in \mathbb{C} : a(\nu) \in \mathbb{R}\}$, *then* $P \subseteq \mathbb{C} \setminus M$.

P r o o f. Denote $a(\nu)h = x$. Since $a(\nu) \in \mathbb{R}$, hence $x \in \mathbb{R}$ and vice versa. For determinant (8), we have a real-valued function on P:

$$\Delta(\nu) = \frac{(e^x - 1)^2 - x^2 e^x}{x^4} h^4.$$

The function $f(x) = \dfrac{(e^x - 1)^2 - x^2 e^x}{x^4}$ has a removable singularity at $x = 0$. Actually,

$$\lim_{x \to 0} f(x) = \lim_{x \to 0} \frac{(e^x - 1)^2 - x^2 e^x}{x^4} =$$
$$= \lim_{x \to 0} \frac{e^{2x} - 2e^x + 1 - x^2 e^x}{x^4} = \left[\frac{0}{0}\right] = \frac{1}{12}.$$

The equation $(e^x - 1)^2 - x^2 e^x = 0$ is equivalent to two equations

$$e^x - 1 - xe^{x/2} = 0 \qquad \text{and} \qquad e^x - 1 + xe^{x/2} = 0,$$

or

$$\operatorname{sh}\frac{x}{2} = \frac{x}{2} \qquad \text{and} \qquad \operatorname{sh}\frac{x}{2} = -\frac{x}{2}.$$

Those equations do not have real nonzero roots. This proves our lemma. ♦

Remark 1. *Basing on Lemma* 1*, we have that the set* $\mathbb{C} \setminus M$ *is not empty, moreover, since the equation* $\operatorname{sh} z = z$ *has nonzero roots in the set of complex numbers* [14]*, hence* $\mathbb{C} \setminus M$ *does not coincide with* $\mathbb{C}$.

2. Main results. In this section, we propose a method of solving the problem (1), (2).

Denote by $x(\lambda)$ the eigenvector of the operator A, which corresponds to its eigenvalue $\lambda \in \mathbb{C}$, i. e. nonzero solutions in H of the equation

$$Ax(\lambda) = \lambda x(\lambda), \qquad \lambda \in \mathbb{C}.$$

If λ is not an eigenvalue of the operator A, then we assume $x(\lambda) = 0$. For the abstract operator $a(A) = \sum_{n=0}^{\infty} a_n A^n$ we have

$$a(A)x(\lambda) = a(\lambda)x(\lambda).$$

Definition 1. We shall say that vector φ from H belongs to $L = L_\Lambda$, where $\Lambda \subseteq \mathbb{C}$, if there exists the following depending on φ pair: a linear operator $R_\varphi(\lambda) : H \to H$ and a measure $\mu_\varphi(\lambda)$, $\lambda \in \Lambda$, such that

$$\varphi = \int_\Lambda R_\varphi(\lambda)x(\lambda)\, d\mu_\varphi(\lambda). \tag{12}$$

Lemma 2. *For arbitrary* $t \in (0,h)$ *and* $\lambda \in \Lambda,\ \Lambda = \mathbb{C} \setminus M$*, the following identities hold:*

$$\left[\frac{d}{dt} - a(A)\right]^2 \{\hat{T}_k(t,\lambda)x(\lambda)\} \equiv 0, \qquad k \in \{1,2\}, \tag{13}$$

where M *is the set* (9) *and* $\hat{T}_1(t,\lambda),\ \hat{T}_2(t,\lambda)$ *are functions* (10) *and* (11) *respectively, analytical in the neighborhood of zero with respect to parameter* $\lambda \in \Lambda$.

P r o o f. Since $\hat{T}_1(t,\lambda)$, $\hat{T}_2(t,\lambda)$ are analytical with respect to parameter $\lambda \in \Lambda$ solutions of equation (3), hence for any $\lambda \in \Lambda$, $t \in (0,h)$ and $k \in \{1,2\}$ we have:

$$\left[\frac{d}{dt} - a(A)\right]^2 \{\hat{T}_k(t,\lambda)x(\lambda)\} = \frac{d^2}{dt^2}\{\hat{T}_k(t,\lambda)x(\lambda)\} - 2\frac{d}{dt}a(A)\{\hat{T}_k(t,\lambda)x(\lambda)\} +$$

$$+ a^2(A)\{\hat{T}_k(t,\lambda)x(\lambda)\} = \frac{d^2}{dt^2}\{\hat{T}_k(t,\lambda)x(\lambda)\} -$$

$$- 2\frac{d}{dt}\{\hat{T}_k(t,\lambda)a(\lambda)x(\lambda)\} + \{\hat{T}_k(t,\lambda)a^2(\lambda)x(\lambda)\} =$$

$$= \left\{\left[\frac{d}{dt} - a(\lambda)\right]^2 \hat{T}_k(t,\lambda)\right\} x(\lambda) \equiv 0.$$

This completes our proof. ♦

Let the vectors φ_1, φ_2 belong to L, i. e. φ_1, φ_2 can be represented in the form

$$\varphi_k = \int_\Lambda R_{\varphi_k}(\lambda)x(\lambda)\, d\mu_{\varphi_k}(\lambda), \qquad k \in \{1,2\}, \tag{14}$$

where $R_{\varphi_1}(\lambda)$, $R_{\varphi_2}(\lambda)$ and $\mu_{\varphi_1}(\lambda)$, $\mu_{\varphi_2}(\lambda)$ are corresponding operators and measures, $\lambda \in \Lambda \subseteq \mathbb{C}$.

Theorem 1. *Let in integral conditions* (2) *the vectors* φ_1, φ_2 *belong to* L, *i.e.* φ_1, φ_2 *can be represented in the form* (14), *where* $\lambda \in \Lambda \equiv \mathbb{C} \setminus M$, M *is set* (9). *Besides, let the following equalities hold:*

$$\left[\frac{d}{dt} - a(A)\right]^2 \int_\Lambda R_{\varphi_k}(\lambda)\{\hat{T}_k(t,\lambda)x(\lambda)\}\, d\mu_{\varphi_k}(\lambda) =$$

$$= \int_\Lambda R_{\varphi_k}(\lambda)\left[\frac{d}{dt} - a(A)\right]^2 \{\hat{T}_k(t,\lambda)x(\lambda)\}\, d\mu_{\varphi_k}(\lambda), \quad k \in \{1,2\}, \tag{15}$$

$$\int_0^h t^s \left(\int_\Lambda R_{\varphi_k}(\lambda)\{\hat{T}_k(t,\lambda)x(\lambda)\}\, d\mu_{\varphi_k}(\lambda) \right) dt =$$

$$= \int_\Lambda R_{\varphi_k}(\lambda)\left\{ \int_0^h t^s \hat{T}_k(t,\lambda)\, dt \right\} x(\lambda)\, d\mu_{\varphi_k}(\lambda),$$

$$s \in \{0,1\}, \quad k \in \{1,2\}. \tag{16}$$

Then the formula

$$U(t) = \sum_{k=1}^{2} \int_\Lambda R_{\varphi_k}(\lambda)\{\hat{T}_k(t,\lambda)x(\lambda)\}\, d\mu_{\varphi_k}(\lambda) \tag{17}$$

defines a solution of problem (1), (2).

P r o o f. According to formulas (15) and (17), we have:

$$\left[\frac{d}{dt} - a(A)\right]^2 U(t) = \left[\frac{d}{dt} - a(A)\right]^2 \sum_{k=1}^{2} \int_\Lambda R_{\varphi_k}(\lambda)\{\hat{T}_k(t,\lambda)x(\lambda)\}\, d\mu_{\varphi_k}(\lambda) =$$

$$= \sum_{k=1}^{2} \int_\Lambda R_{\varphi_k}(\lambda)\left[\frac{d}{dt} - a(A)\right]^2 \{\hat{T}_k(t,\lambda)x(\lambda)\}\, d\mu_{\varphi_k}(\lambda).$$

From the identities (13), we obtain

$$\left[\frac{d}{dt} - a(A)\right]^2 U(t) = \sum_{k=1}^{2} \int_\Lambda R_{\varphi_k}(\lambda)\{0\}\, d\mu_{\varphi_k}(\lambda).$$

Since the operators $R_{\varphi_1}(\lambda)$, $R_{\varphi_2}(\lambda)$ are linear, the last integrals are equal to zero in H, i.e. $U(t)$ satisfies equation (1). Basing on equalities (5), (6), and on formulas (16) and (17), we shall prove the realization of first integral condition in (2):

$$\int_0^h U(t)\, dt = \int_0^h \left(\sum_{k=1}^{2} \int_\Lambda R_{\varphi_k}(\lambda)\{\hat{T}_k(t,\lambda)x(\lambda)\}\, d\mu_{\varphi_k}(\lambda) \right) dt =$$

$$= \sum_{k=1}^{2} \int_\Lambda R_{\varphi_k}(\lambda)\left\{ \int_0^h \hat{T}_k(t,\lambda)\, dt \right\} x(\lambda)\, d\mu_{\varphi_k}(\lambda) =$$

$$= \int_\Lambda R_{\varphi_1}(\lambda)\{1 \cdot x(\lambda)\}\, d\mu_{\varphi_1}(\lambda) +$$

$$+ \int_\Lambda R_{\varphi_2}(\lambda)\{0 \cdot x(\lambda)\}\, d\mu_{\varphi_2}(\lambda) = \varphi_1.$$

By analogy, we can prove the realization of the second integral condition in (2). This completes our proof. ♦

3. Problem with integral conditions for partial differential equation. In this section, we shall give the example of using an abstract approach to solving the problem in the strip $\{(t,x): t \in (0,h), x \in \mathbb{R}\}$ for the partial differential equation

$$\left[\frac{\partial}{\partial t} - a\left(\frac{\partial}{\partial x}\right)\right]^2 U(t,x) = 0, \qquad t \in (0,h), \qquad x \in \mathbb{R}, \tag{18}$$

with integral conditions

$$\int_0^h U(t,x)\,dt = \varphi_1(x), \qquad \int_0^h tU(t,x)\,dt = \varphi_2(x), \qquad x \in \mathbb{R}. \tag{19}$$

We shall represent this problem as problem (1), (2), in which $A = \frac{d}{dx}$, $e^{\lambda x}$ is an eigenvector (eigenfunction) of the operator A for $\lambda \in \mathbb{C}$, H is a class of entire functions, $L = K_{\mathbb{C}\setminus M}$ is a linear class of quasipolynomials of the form

$$\varphi(x) = \sum_{j=1}^{m} Q_j(x) e^{\alpha_j x}, \tag{20}$$

where $Q_1(x), \dots, Q_m(x)$ are polynomials with complex coefficients, $\alpha_j \in \mathbb{C} \setminus M$, $j \in \{1,\dots,m\}$, $\alpha_j \neq \alpha_k$ for $j \neq k$, $x \in \mathbb{R}$, $m \in \mathbb{N}$, M is set (9).

As a measure $\mu(\lambda)$, we take the Dirac measure. From representation (12), we obtain

$$\varphi(x) = \int_\Lambda R_\varphi(\lambda) e^{\lambda x}\, d\mu_\varphi(\lambda) = R_\varphi(\lambda) e^{\lambda x}\Big|_{\lambda=0},$$

from which it follows that

$$R_\varphi(\lambda) e^{\lambda x} = \varphi\left(\frac{d}{d\nu}\right) e^{\nu x}\Big|_{\nu=\lambda}.$$

Each quasipolynomial $\varphi(x)$ of the form (20) defines [4] differential operations $Q_j\left(\frac{d}{d\lambda}\right)$, $j \in \{1,\dots,m\}$, of finite order on the class of entire functions $\Phi(\lambda)$, namely

$$\varphi\left(\frac{d}{d\lambda}\right)\Phi(\lambda) = \sum_{j=1}^{m} Q_j\left(\frac{d}{d\lambda}\right)\Phi(\lambda + \alpha_j),$$

in particular,

$$\varphi\left(\frac{d}{d\lambda}\right)\Phi(\lambda)\Big|_{\lambda=0} = \sum_{j=1}^{m} Q_j\left(\frac{d}{d\lambda}\right)\Phi(\lambda)\Big|_{\lambda=\alpha_j}.$$

From formula (17), for quasipolynomials $\varphi_k(x) = \sum_{j=1}^{m} Q_{jk}(x) e^{\alpha_{jk} x}$, $k \in \{1,2\}$, we obtain the representation of the solution of problem (18), (19) in the form

$$U(t,x) = \sum_{k=1}^{2} \varphi_k\left(\frac{d}{d\lambda}\right)\{\hat{T}_k(t,\lambda) e^{\lambda x}\}\Big|_{\lambda=0} =$$

$$= \sum_{k=1}^{2}\sum_{j=1}^{m} Q_{kj}\left(\frac{d}{d\lambda}\right)\{\hat{T}_k(t,\lambda) e^{\lambda x}\}\Big|_{\lambda=\alpha_{kj}}. \tag{21}$$

One can easily prove that if $\varphi_1, \varphi_2 \in K_{\mathbb{C}\setminus M}$, where M is set (9), then the solution (21) of problem (18), (19) exists and is unique in the appropriate class of quasipolynomials of variables t, x.

Conclusions. In this work, we propose a method of solving a problem with inhomogeneous integral conditions for homogeneous differential-operator equation with abstract operator in a linear space. The solution of the problem is represented in the form of Stieltjes integrals with respect to certain measures. We give the example of applying this method to solving the ill-posed problem with integral conditions for the second order partial differential equation in time variable, in which the integral conditions are given, and, in general, an infinite order partial differential equation in a spatial variable. In the future research, the subject of interest is the development of an analogous method of solving the problem with integral conditions for inhomogeneous differential-operator equation.

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ЗАДАЧА З ІНТЕГРАЛЬНИМИ УМОВАМИ ДЛЯ ДИФЕРЕНЦІАЛЬНО-ОПЕРАТОРНОГО РІВНЯННЯ

Запропоновано метод розв'язування задачі з неоднорідними інтегральними умовами для однорідного диференціально-операторного рівняння з абстрактним оператором у лінійному просторі H. *Для правих частин інтегральних умов, що належать до спеціального підпростору* $L \subseteq H$, *у якому вектори зображаються у вигляді інтегралів Стілтьєса за деякими мірами, розв'язок задачі подано у вигляді інтегралів Стілтьєса за цими ж мірами. Наведено приклад застосування методу до розв'язування некоректної задачі для рівняння з частинними похідними другого порядку за часовою змінною (за якою задано інтегральні умови) та в загальному нескінченного порядку за просторовою змінною.*

ЗАДАЧА С ИНТЕГРАЛЬНЫМИ УСЛОВИЯМИ ДЛЯ ДИФФЕРЕНЦИАЛЬНО-ОПЕРАТОРНОГО УРАВНЕНИЯ

Предложен метод решения задачи с неоднородными интегральными условиями для однородного дифференциально-операторного уравнения с абстрактным оператором в линейном пространстве H. *Для правых частей интегральных условий, принадлежащих специальному подпространству* $L \subseteq H$, *в котором векторы представляются в виде интегралов Стилтьеса по некоторым мерам, решение задачи представлено в виде интегралов Стилтьеса по этим же мерам. Приведен пример использования метода к решению некорректной задачи для уравнения в частных производных второго порядка по временной переменной (по которой заданы интегральные условия) и бесконечного порядка по пространственной переменной.*

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