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LONGITUDINAL WAVES PROPAGATION IN PLATES IN THE PRESENCE OF TRANSVERSAL MAGNETIC FIELD

The problem on longitudinal wave propagation in a plate in the presence of a constant transversal magnetic field is studies. The asymptotic behavior of tangential displacement of the points of the plate median surface is found. It is established that the wave of a given initial shape finally turns into a quasi-harmonic one.

The works [1–3, 5, 6] are devoted to study of processes of vibration and wave propagation in electroconductive bodies. In proposal work a problem of longitudinal wave propagation in plates in presence of external transversal magnetic field is investigated.

1. Let the thin elastic layer (the plate) with $2h$ constant thickness, with σ limited electroconductivity is situated in external magnetic field with $\mathbf{B}(0,\,0,\,B_{03})$ constant vector of magnetic induction. In case of plane problem (wave propagation is independent from y coordinate) for longitudinal waves we have the following system of equations [1]:

$$2h\,\frac{E}{1-\nu^2}\,\partial_x^2 U + 2\sigma h\,\frac{1}{c}\left(B_{03}\Psi - B_{03}^2\,\frac{1}{c}\,\partial_t U\right) = 2\rho h \partial_t^2 U\,,$$

$$\partial_x f + 4\pi\sigma\,\frac{1}{c}\left(\Psi - B_{03}\,\frac{1}{c}\,\partial_t U\right) = \frac{1}{2h}\,(h_1^+ - h_1^-)\,,$$

$$\partial_x \Psi + \frac{1}{c}\,\partial_t f = 0\,,$$

$$\left(\partial_x^2 - \frac{1}{c^2}\,\partial_t^2\right)(h_1^+ - h_1^-) = 2\,\frac{1}{\lambda}\left(\partial_x f + \frac{1}{c}\,\partial_t \Psi\right), \qquad (1)$$

where E, ρ, ν are the elastic characteristics of plate, $U(x,t)$ – the displacements of points of middle plane of plate, c – the electrodynamical constant, $\Psi(x,t)$ and $f(x,t)$ – the unknown functions of the perturbed electromagnetic field, $h_1^{\pm}$ – the components of the perturbed magnetic field on the surfaces of the plate, λ – a typical size for that problem (the length of half-wave of plate elastic vibration).

System (1) is closed system concerning unknown functions U, f, Ψ, h_1^+, h_1^-. From this system we can determine the velocity of longitudinal waves in presence of magnetic field.

The solution of (1) system is introduced as running waves, which are propagate along plate

$$Q = Q_0 \exp\left(i(\omega t - kx)\right), \qquad Q_0 = \text{const}\,, \qquad (2)$$

where ω is the frequency of vibrations, k – the wave number.

Substituting (2) into (1) and letting $\lambda = 1/k$ [2], for determination of phase velocity we get the following characteristic equation:

$$\frac{\Omega^3}{k^3} + \frac{c^2 k}{4\pi\sigma}\,\frac{1+kh}{kh}\,\frac{\Omega^2}{k^2} + \left(\frac{B_{03}^2}{4\pi\rho}\,\frac{1+kh}{kh} + \frac{E}{\rho(1-\nu^2)}\right)\frac{\Omega}{k} + \frac{c^2 k}{4\pi\sigma}\,\frac{1+kh}{kh}\,\frac{E}{\rho(1-\nu^2)} = 0, \qquad i\omega = \Omega\,. \qquad (3)$$

For ideal conductive plate from (3) by $\sigma \to \infty$ we have the following dispersion equation:

$$\frac{\omega^2}{k^2} = \frac{E}{\rho(1-\nu^2)} + \frac{B_{03}^2}{4\pi\rho}\frac{1+kh}{kh}. \tag{4}$$

From equations (3) and (4) it is seen that the presence of magnetic field causes dispersion: the phase velocity is dependent from kh parameter. In absence of magnetic field ($B_{03} = 0$), the phase velocity is equal to velocity of elastic longitudinal wave propagation. With increasing of $1/kh$, the phase velocity increase and magnetic field can cause an increase of phase velocity about 2.5 times [7].

2. The equation (4) was obtained from (3) by $\sigma \to \infty$. The equation (4) can be obtained also directly from governing equations of the problem. For this purpose determine Ψ from second equation of system (1) and substitute it into the other equations.

By $\sigma \to \infty$ and noting that $B_{03}\frac{1}{c^2}\partial_t^2 U \ll \partial_x f$, we get the following system of equations:

$$\partial_t^2 U - \frac{E}{\rho(1-\nu^2)}\partial_x^2 U + \frac{B_{03}^2}{4\pi\rho}\partial_x f - \frac{B_{03}}{8\pi\rho h}(h_1^+ - h_1^-) = 0,$$

$$B_{03}\partial_{xt}^2 U + \partial_t f = 0, \qquad \partial_x^2(h_1^+ - h_1^-) = 2\frac{1}{\lambda}\partial_x f. \tag{5}$$

Indeed substituting the (2) to the system of equations (5), we get the dispersion equation (4).

Now let's get the values $U(x,t)$ under the following initial conditions:

$$U(x,0) = \beta(x), \qquad \partial_t U(x,0) = \gamma(x), \qquad f(x,0) = 0. \tag{6}$$

The boundary conditions in this case are the followings: all of functions and then derivatives with respect to x is equal to 0 at $x = \pm\infty$. Applying the Laplace's right-side transformation by the variable t and Fourier's transformation by the variable x on the system of equations (5), with account of (6) conditions, we get:

$$\bar{\bar{U}}(\alpha,p) = \left[\gamma^*(\alpha) + \beta^*(\alpha)\left(p + \alpha^2 B_{03}^2 \frac{1+\alpha h}{p\alpha h}\frac{1}{4\pi\rho}\right)\right]\left[p^2 + \right.$$
$$\left. + \alpha^2 \frac{E}{\rho(1-\nu^2)} + B_{03}^2 \frac{1}{4\pi\rho}\frac{1+\alpha h}{\alpha h}\right]^{-1}. \tag{7}$$

Here $\bar{\bar{U}}(\alpha,p)$ is the Fourier's transforming of $\bar{U}(x,p)$ function, $\bar{U}(x,p)$ – the Laplace's transforming of $U(x,t)$ function,

$$\beta^*(\alpha) = \int_{-\infty}^{+\infty} \beta(x)\exp(i\alpha x)\,dx, \qquad \gamma^*(\alpha) = \int_{-\infty}^{+\infty} \gamma(x)\exp(i\alpha x)\,dx.$$

For determination of $U(x,t)$ it is necessary to make the opposite transformation of Laplace and Fourier in (7). Consider the case when $\alpha h \ll 1$ and at $t = 0$ we have $\beta(x) = 0$, $\gamma(x) = \gamma_0\delta(x)$ ($\gamma_0 = \text{const}$, $\delta(x)$ is the Dirac's function). If magnetic field is absent then from (7) we have

$$U(x,t) = \gamma_0\left(\frac{\rho(1-\nu^2)}{E}\right)^{1/2}\begin{cases} \frac{1}{2}, & \frac{x}{t} < \left(\frac{E}{\rho(1-\nu^2)}\right)^{1/2}, \\ \frac{1}{4}, & \frac{x}{t} = \left(\frac{E}{\rho(1-\nu^2)}\right)^{1/2}, \\ 0, & \frac{x}{t} > \left(\frac{E}{\rho(1-\nu^2)}\right)^{1/2}. \end{cases} \tag{8}$$

As one can from (8) we haven't dispersion. From (7) in presence of magnetic field for $U(x,t)$ we get

$$U(x,t) = \frac{\gamma_0}{2\pi} \int\limits_{-\infty}^{+\infty} \left(\alpha^2 \frac{E}{\rho(1-\nu^2)} + \alpha \frac{B_{03}^2}{4\pi\rho h} \right)^{-1/2} \times$$
$$\times \sin\left[t \left(\alpha^2 \frac{E}{\rho(1-\nu^2)} + \alpha \frac{B_{03}^2}{4\pi\rho h} \right)^{1/2} \right] \exp(-i\alpha x)\, d\alpha\,. \qquad (9)$$

The presence of magnetic field leads to that to the following: the media has a dispersion, i.e. w is a nonlinear function of k. In this case it is impossible to calculate the integral (9) exactly. Applying the method of stationar phase on the (9) integral large values of t, and fixed x/t for $U(x,t)$ we get the following asymptotic behavior:

$$U(x,t) \approx -\frac{\gamma_0}{2} \frac{1}{(\pi s_1)^{1/2}} \frac{1}{(x^2 - s_2 t^2)^{1/4}} \sin(k_0 x - \omega_0 t + 0.25\pi)\,,$$

where $$k_0 = \frac{s_1}{2 s_2}\left[x - (x^2 - s_2 t^2)^{1/2}\right] \frac{1}{(x^2 - s_2 t^2)^{1/2}}\,, \quad \omega_0 = \frac{s_1}{2} t \frac{1}{(x^2 - s_2 t^2)^{1/2}}\,,$$
$$s_1 = B_{03}^2 \frac{1}{4\pi\rho h}\,, \qquad s_2 = E \frac{1}{\rho(1-\nu^2)}\,.$$

So, we obtain that the wave of origin form $U\big|_{t=0} = 0,\ \partial_t U\big|_{t=0} = \gamma_0 \delta(x)$ finally converts to quasiharmonic wave (with 0.25π phase), which has a wave number k_0 and ω_0 frequency, dependent on x/t. The amplitude of this wave is depend on x/t and damping proportionally to $1/\sqrt{t}$. In the neighborhood of the given value of x and by a fixed value of t, we may consider the k_0 and ω_0 as constants [4].

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ПОШИРЕННЯ ПОЗДОВЖНІХ ХВИЛЬ У ПЛАСТИНАХ ПРИ НАЯВНОСТІ ПОПЕРЕЧНОГО МАГНІТНОГО ПОЛЯ

Досліджується задача про поширення поздовжньої хвилі в пластині при наявності постійного поперечного магнітного поля. Знайдено асимптотичну поведінку тангенціального переміщення точок серединної площини пластинки. Встановлено, що хвиля із заданою початковою формою в кінцевому рахунку перетворюється в квазігармонічну хвилю.

РАСПРОСТРАНЕНИЕ ПРОДОЛЬНЫХ ВОЛН В ПЛАСТИНАХ ПРИ НАЛИЧИИ ПОПЕРЕЧНОГО МАГНИТНОГО ПОЛЯ

Исследуется задача о распространении продольной волны в пластине при наличии постоянного поперечного магнитного поля. Найдено асимптотическое поведение тангенциального перемещения точек срединной плоскости пластинки. Установлено, что волна с заданной начальной формой в конечном счете превращается в квазигармоническую волну.

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